

NUMBER

Fractions, decimals, percentages, ratio and proportion

Pupils should learn to:

As outcomes, Year 7 pupils should, for example:

Calculate fractions of quantities; add, subtract, multiply and divide fractions

Add and subtract simple fractions.

Know addition facts for simple fractions, such as:

- $\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$
- $\frac{3}{4} + \frac{3}{4} = 1\frac{1}{2}$
- $\frac{1}{8} + \frac{1}{8} = \frac{1}{4}$

and derive other totals from these results, such as:

- $\frac{1}{4} + \frac{1}{8} = \frac{3}{8}$ (knowing that $\frac{1}{4} = \frac{2}{8}$)
- $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{1}{2}$ (knowing that $\frac{4}{8} = \frac{1}{2}$)

Add and subtract simple fractions with the same denominator.

For example:

- $\frac{3}{8} + \frac{5}{8}$ $\frac{3}{5} + \frac{4}{5} + \frac{1}{5}$ $\frac{7}{10} + \frac{3}{10} + \frac{5}{10} + \frac{8}{10}$
- $\frac{6}{7} - \frac{4}{7}$ $\frac{9}{10} + \frac{4}{10} - \frac{3}{10}$

Calculate fractions of numbers, quantities or measurements.

Know that, for example:

- $\frac{1}{5}$ of 35 has the same value as $35 \div 5 = 7$;
- $\frac{2}{3}$ of 15 has the same value as $15 \div 3 \times 2 = 10$;
- 0.5 of 18 has the same value as $\frac{1}{2}$ of 18 = 9.

Use mental methods to answer short questions with whole-number answers, such as:

- Find: one fifth of 40; two thirds of 150 g.
- Find: $\frac{1}{3}$ of 24; $\frac{3}{8}$ of 160; $\frac{9}{10}$ of 1 metre.
- Find: 0.5 of 50; 0.75 of 56; 1.25 of 40.

Use informal written methods to answer questions such as:

- If I make one fifth of a turn, how many degrees do I turn?
- Calculate: $\frac{7}{10}$ of £420; $\frac{6}{5}$ of 35;
 $\frac{3}{7}$ of 210; $1\frac{1}{4}$ of 2.4.

Link to multiplying fractions (pages 68–9).

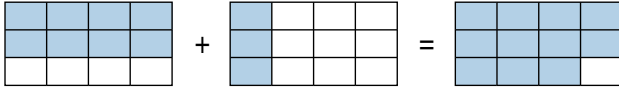
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As outcomes, Year 8 pupils should, for example:

Add and subtract fractions.

Use diagrams to illustrate adding and subtracting fractions, showing equivalence.



Know that fractions can only be added and subtracted if they have the same denominator.

Use, for example, a single bar to avoid the problem of adding denominators:

$$\frac{3}{5} + \frac{2}{5} = \frac{6+2}{10} = \frac{8}{10} = 1 \frac{1}{10}$$

Answer questions such as:

- Add/subtract these fractions:
 $\frac{1}{4} + \frac{5}{12}$ $\frac{3}{5} + \frac{3}{4}$ $\frac{5}{6} - \frac{3}{4}$
- Ancient Egyptian fractions were written with 1 as the numerator (unit fractions). Express these fractions as sums of unit fractions:
 $\frac{5}{8}$, $1\frac{1}{12}$, $\frac{7}{10}$, $\frac{7}{12}$, $\frac{9}{20}$
- This fraction sum is made from four different digits, 1, 2, 4 and 8. The fraction sum is 1.

$$\frac{1}{2} + \frac{4}{8}$$

Find other fraction sums made from four different digits and with a fraction sum of 1.

Calculate fractions of numbers, quantities or measurements.

Develop written methods to answer short questions with fraction answers, such as:

- Find: three fifths of 17;
two thirds of 140 g;
 $\frac{6}{25}$ of 34.

Link to multiplying and dividing fractions (pages 68–9).

As outcomes, Year 9 pupils should, for example:

Add and subtract fractions.

Add and subtract more complex fractions. For example:

- $\frac{11}{18} + \frac{7}{24} = \frac{44+21}{72} = \frac{65}{72}$
- A photograph is $6\frac{1}{4}$ inches tall and $8\frac{5}{8}$ inches wide. Calculate its perimeter.
- *Investigate $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$ and similar series.*
- *Begin to add and subtract algebraic fractions (pages 118–19), linking to number examples.*

Link to finding lowest common multiples (pages 54–5).

Calculate fractions of numbers, quantities or measurements.

Understand the multiplicative nature of fractions as operators. For example:

- Which is the greater: $\frac{3}{4}$ of 24 or $\frac{2}{3}$ of 21?
34% of 75 or $\frac{5}{7}$ of 85?
5 out of 16 or 8 out of 25?
- In a survey of 24 pupils, $\frac{1}{3}$ liked football best, $\frac{1}{4}$ liked basketball, $\frac{3}{8}$ liked athletics and the rest liked swimming. How many liked swimming?
- Brian used $\frac{1}{3}$ of a 750 g bag of flour to make scones. Claire used $\frac{2}{5}$ of the flour that remained to make a cake. How many grams of flour were left in the bag?
- In a bag of 20 coloured beads, $\frac{2}{5}$ are red, $\frac{1}{4}$ are blue, $\frac{1}{10}$ are yellow and 3 are green. The rest are black. What fraction are black?

Link to mutually exclusive events in probability (pages 278–81).

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Fractions, decimals, percentages, ratio and proportion

Pupils should learn to:

As outcomes, Year 7 pupils should, for example:

Calculate fractions of quantities; add, subtract, multiply and divide fractions (continued)

Multiply a fraction by an integer or an integer by a fraction.

Know that $\frac{1}{4}$ of 12, $\frac{1}{4} \times 12$, $12 \times \frac{1}{4}$ and $12 \div 4$ are all equivalent.

Multiply a simple fraction by an integer. For example:

$$\frac{1}{5} \times 3 = \frac{3}{5} \qquad \frac{2}{5} \times 4 = \frac{8}{5}$$

Simplify the product of a simple fraction and an integer.

For example:

$$\frac{1}{5} \times 15 = 3$$

$$\frac{2}{5} \times 15 = 2 \times \frac{1}{5} \times 15 = 2 \times 3 = 6$$

$$12 \times \frac{5}{6} = \frac{5}{6} \times 12 = 5 \times \frac{1}{6} \times 12 = 5 \times 2 = 10$$

Answer questions such as:

- Find: $\frac{1}{9} \times 63$ $\frac{7}{9} \times 90$ $1\frac{1}{4} \times 10$
- Find: 0.25×24 0.2×50 3.3×40

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As outcomes, Year 8 pupils should, for example:

Multiply an integer by a fraction.

Know that $\frac{2}{3}$ of 12, $\frac{2}{3} \times 12$ and $12 \times \frac{2}{3}$ are all equivalent.

Connect ordinary multiplication tables with patterns in fraction multiplication tables:

$$\begin{array}{lll} \frac{1}{5} \times 1 = \frac{1}{5} & \frac{2}{5} \times 1 = \frac{2}{5} & \frac{3}{5} \times 1 = \frac{3}{5} \\ \frac{1}{5} \times 2 = \frac{2}{5} & \frac{2}{5} \times 2 = \frac{4}{5} & \frac{3}{5} \times 2 = \frac{6}{5} \\ \frac{1}{5} \times 3 = \frac{3}{5} & \frac{2}{5} \times 3 = \frac{6}{5} & \frac{3}{5} \times 3 = \frac{9}{5} \\ \frac{1}{5} \times 4 = \frac{4}{5} & \frac{2}{5} \times 4 = \frac{8}{5} & \frac{3}{5} \times 4 = \frac{12}{5} \end{array}$$

Think of multiplication by $\frac{1}{8}$ as division by 8, so $6 \times \frac{1}{8} = 6 \div 8$, and $6 \times \frac{3}{8} = 6 \times 3 \times \frac{1}{8} = 18 \div 8$.

Use cancellation to simplify the product of a fraction and an integer. For example:

$$\frac{7}{24} \times \frac{15}{1} = \frac{7}{\cancel{24}^8} \times \frac{\cancel{15}^5}{1} = \frac{35}{8}$$

Answer questions such as:

- Find: $\frac{3}{12} \times 30$ $\frac{5}{9} \times 24$ $2\frac{1}{8} \times 10$

Understand that when multiplying a positive number by a fraction less than one, the result will be a smaller number. For example:

$$24 \times \frac{1}{4} = 6$$

Divide an integer by a fraction.

Know that a statement such as $24 \div \frac{1}{4}$ can be interpreted as:

- How many quarters are there in 24?
 $24 = \square \times \frac{1}{4}$ or $24 = \frac{1}{4} \times \square$.

For example:

- Look at one whole circle (or rectangle, prism...). How many sevenths can you see? (Seven.)
- Look at 1. How many fifths can you see? (Five.)
- Look at 4. How many fifths can you see? (Twenty.)
- Look at 4. How many two fifths can you see? (Ten.)

Use patterns. For example:

$$\begin{array}{ll} 60 \times \frac{1}{6} = 10 & \text{and} \quad 10 \div \frac{1}{6} = 60 \\ 30 \times \frac{2}{6} = 10 & \text{and} \quad 10 \div \frac{2}{6} = 30 \\ 20 \times \frac{3}{6} = 10 & \text{and} \quad 10 \div \frac{3}{6} = 20 \\ 15 \times \frac{4}{6} = 10 & \text{and} \quad 10 \div \frac{4}{6} = 15 \\ 12 \times \frac{5}{6} = 10 & \text{and} \quad 10 \div \frac{5}{6} = 12 \end{array}$$

Understand that when dividing a positive number by a fraction less than one, the result will be a larger number.

For example:

$$24 \div \frac{1}{4} = 96$$

As outcomes, Year 9 pupils should, for example:

Multiply a fraction by a fraction.

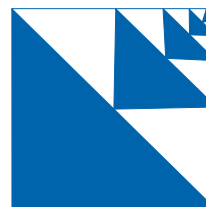
Multiply fractions, using cancelling to simplify:

$$\frac{3}{4} \times \frac{2}{9} = \frac{\cancel{3}^1 \times \cancel{2}^1}{\cancel{4}^2 \times \cancel{9}^3} = \frac{1}{6}$$

For example:

- Calculate:
 - a. $\frac{3}{5} \times \frac{20}{33} \times \frac{22}{14}$
 - b. $\frac{22}{7} \times 14 \times 14$
 - c. $4\frac{2}{3} \times 1\frac{3}{4}$
 - d. $\frac{1}{2}(2 - \frac{1}{4})$
 - e. $(2\frac{1}{2})^3$
- A photograph is $6\frac{1}{4}$ inches tall and $8\frac{5}{8}$ inches wide. Calculate its area.

- Imagine a square with sides of 1 metre. The area of the largest shaded triangle is $\frac{1}{2} \text{ m}^2$.



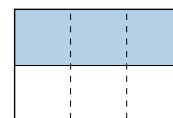
- Write the areas of the next two largest shaded triangles.
- Use the diagram to help you find the sum of the infinite series:
 $\frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \frac{1}{128} + \dots$
Explain how you arrived at your solution.

Divide a fraction by a fraction.

Use the inverse rule to divide fractions, first converting mixed numbers to improper fractions.

For example:

- Look at one half of a shape.



How many sixths of the shape can you see? (Six.)
So, how many sixths in one half? (Three.)
So $\frac{1}{2} \div \frac{1}{6} = \frac{1}{2} \times \frac{6}{1} = \frac{6}{2} = 3$

- $\frac{2}{3} \div \frac{4}{7} = \frac{2}{3} \times \frac{7}{4} = \frac{14}{12}$ or $\frac{7}{6}$
- $2\frac{1}{3} \div \frac{4}{5} = \frac{7}{3} \times \frac{5}{4} = \frac{35}{12}$ or $2\frac{11}{12}$

Answer questions such as:

- Calculate: $(1 - \frac{1}{3}) / (1 - \frac{5}{6})$
- The area of a circle is 154 cm^2 . Taking π as $\frac{22}{7}$, find the radius of the circle.